Errata

for

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Algorithm 7.2 Building a SIFT scale space. This procedure is an extension of Alg. 7.1 and takes the same parameters. The SIFT scale space consists of two components: a hierarchical Gaussian scale space $\mathbf{G} = (\mathbf{G}_0, \ldots, \mathbf{G}_{P-1})$ and a hierarchical DoG scale space $\mathbf{D} = (\mathbf{D}_0, \ldots, \mathbf{D}_{P-1})$. Each Gaussian octave $\mathbf{G}_p$ holds $Q+3$ levels $(\mathbf{G}_{p,-1}, \ldots, \mathbf{G}_{p,Q+1})$. For each Gaussian octave, the first level $\mathbf{G}_{p,-1}$ is obtained by decimating level $\mathbf{G}_{p-1,Q}$ of the previous octave. A DoG octave $\mathbf{D}_p$ contains $Q+2$ levels $(\mathbf{D}_{p,-1}, \ldots, \mathbf{D}_{p,Q})$. Each DoG level $\mathbf{D}_{p,q}$ is calculated as the pointwise difference of two adjacent Gaussian levels $\mathbf{G}_{p,q+1}$ and $\mathbf{G}_{p,q}$. Typical settings are $\sigma_\mathrm{s} = 0.5$, $\sigma_0 = 1.6$, $Q = 3$, $P = 4$.

BuildSiftScaleSpace$(I, \sigma_\mathrm{s}, \sigma_0, P, Q)$
Input: I, source image; σ_s, sampling scale; σ_0, reference scale of the first octave; P, number of octaves; Q, number of scale steps per octave; Returns a SIFT scale space representation $\langle \mathbf{G}, \mathbf{D} \rangle$ of the image I.

$\sigma_{0,-1} \leftarrow \sigma_0 \cdot 2^{-1/Q}$ ▷ abs. scale at level $(0,-1)$, Eqn. (7.48)
$\hat{\sigma}_{0,-1} \leftarrow (\sigma_{0,-1}^2 - \sigma_\mathrm{s}^2)^{1/2}$ ▷ relative scale w.r.t. σ_s, Eqn. (7.49)
$\mathbf{G}_{0,-1} \leftarrow I * H^{\mathrm{G},\hat{\sigma}_{0,-1}}$ ▷ apply 2D Gaussian filter of width $\hat{\sigma}_{0,-1}$

$\mathbf{G}_0 \leftarrow$ MakeGaussianOctave$(\mathbf{G}_{0,-1}, 0)$ ▷ create Gauss. octave 0
for $p \leftarrow 1, \ldots, P-1$ **do** ▷ for octaves $1, \ldots, P-1$
  $\mathbf{G}_{p-1,Q} \leftarrow \mathbf{G}_{p-1}(Q)$
  $\mathbf{G}_{p,-1} \leftarrow$ Decimate$(\mathbf{G}_{p-1,Q-1})$ ▷ see Alg. 7.1
  $\mathbf{G}_p \leftarrow$ MakeGaussianOctave$(\mathbf{G}_{p,-1}, p)$ ▷ create octave p

$\mathbf{G} \leftarrow (\mathbf{G}_0, \ldots, \mathbf{G}_{P-1})$ ▷ assemble the Gaussian scale space $\mathbf{G}$

for $p \leftarrow 0, \ldots, P-1$ **do**
  $\mathbf{D}_p \leftarrow$ MakeDogOctave$(\mathbf{G}_p, p)$

$\mathbf{D} \leftarrow (\mathbf{D}_0, \ldots, \mathbf{D}_{P-1})$ ▷ assemble the DoG scale space $\mathbf{D}$
return $\langle \mathbf{G}, \mathbf{D} \rangle$

MakeGaussianOctave$(\mathbf{G}_{p,-1}, p)$ ▷ Gaussian base level $\mathbf{G}_{p,-1}$
for $q \leftarrow 0, \ldots, Q+1$ **do**
  $\tilde{\sigma}_q \leftarrow \sigma_0 \cdot \sqrt{2^{2q/Q} - 2^{-2/Q}}$ ▷ rel. scale w.r.t base level $\mathbf{G}_{p,-1}$
  $\mathbf{G}_{p,q} \leftarrow \mathbf{G}_{p,-1} * H^{\mathrm{G},\tilde{\sigma}_q}$ ▷ apply 2D Gaussian filter of width $\tilde{\sigma}_q$
return $(\mathbf{G}_{p,-1}, \ldots, \mathbf{G}_{p,Q+1})$. ▷ Gaussian octave $\mathbf{G}_p$

MakeDogOctave$(\mathbf{G}_p, p)$ ▷ Gaussian octave $\mathbf{G}_p$
for $q \leftarrow -1, \ldots, Q$ **do**
  $\mathbf{G}_{p,q} \leftarrow \mathbf{G}_p(q)$
  $\mathbf{G}_{p,q+1} \leftarrow \mathbf{G}_p(q+1)$
  $\mathbf{D}_{p,q} \leftarrow \mathbf{G}_{p,q+1} - \mathbf{G}_{p,q}$ ▷ diff. of Gaussians, Eqn. (7.31)
return $(\mathbf{D}_{p,-1}, \mathbf{D}_{p,0}, \ldots, \mathbf{D}_{p,Q})$. ▷ DoG octave $\mathbf{D}_p$

which performs a rotation by the dominant orientation θ and maps the original (rotated) square of size $w_\mathrm{d} \times w_\mathrm{d}$ to the unit square with coordinates $u', v' \in [-0.5, +0.5]$ (see Fig. 7.24).

To make feature vectors rotation-invariant, the individual gradient orientations $\phi(u, v)$ (Eqn. (7.86)) are rotated by

$$\phi'(u, v) = (\phi(u, v) - \theta) \bmod 2\pi, \tag{7.90}$$

such that $\phi'(u, v) \in [0, 2\pi)$, to align with the reference orientation θ. For each gradient sample, with the continuous coordinates (u', v', ϕ'), the corresponding quantity $z(u, v)$ (Eqn. (7.87)) is accumulated into the three-dimensional gradient histogram h_∇. For a complete description of this step see procedure UpdateGradientHistogram() in Alg. 7.9. It first maps the coordinates (u', v', ϕ') (see Eqn. (7.89)) to the continuous histogram position (i', j', k') by

$$\begin{aligned} i' &= \mathsf{n}_\mathrm{spat} \cdot u' + 0.5 \cdot (\mathsf{n}_\mathrm{spat} - 1), \\ j' &= \mathsf{n}_\mathrm{spat} \cdot v' + 0.5 \cdot (\mathsf{n}_\mathrm{spat} - 1), \\ k' &= \phi' \cdot \frac{\mathsf{n}_\mathrm{angl}}{2\pi}, \end{aligned} \tag{7.91}$$

such that $i', j' \in [-0.5, \mathsf{n}_\mathrm{spat} - 0.5]$ and $k' \in [0, \mathsf{n}_\mathrm{angl})$.

Analogous to inserting into a continuous position of a one-dimensional histogram by linear interpolation over *two* bins (see Fig. 7.18), the quantity z is distributed over *eight* neighboring histogram bins by *tri-linear* interpolation. The quantiles of z contributing to the individual histogram bins are determined by the distances of the coordinates (i', j', k') from the discrete indexes (i, j, k) of the affected histogram bins. The bin indexes are found as the possible combinations (i, j, k), with $i \in \{i_0, i_1\}$, $j \in \{j_0, j_1\}$, $k \in \{k_0, k_1\}$, and

$$\begin{aligned} i_0 &= \lfloor i' \rfloor, & i_1 &= (i_0 + 1), \\ j_0 &= \lfloor j' \rfloor, & j_1 &= (j_0 + 1), \\ k_0 &= \lfloor k' \rfloor \bmod \mathsf{n}_\mathrm{angl}, & k_1 &= (k_0 + 1) \bmod \mathsf{n}_\mathrm{angl}. \end{aligned} \tag{7.92}$$

The corresponding quantiles (weights) are

$$\begin{aligned} \alpha_0 &= \lfloor i' \rfloor + 1 - i' = i_1 - i', & \alpha_1 &= 1 - \alpha_0, \\ \beta_0 &= \lfloor j' \rfloor + 1 - j' = j_1 - j', & \beta_1 &= 1 - \beta_0, \\ \gamma_0 &= \lfloor k' \rfloor + 1 - k', & \gamma_1 &= 1 - \gamma_0, \end{aligned} \tag{7.93}$$

and the 8 affected bins of the gradient histogram h_∇ are finally updated as